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On Dispositions

The Solution of the Problem of Defining Dispositional Concepts

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I. Remarks

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One of the main problems of Analytic Philosophy in general and Philosophy of Science in particular consists in how to define dispositional concepts. In sciences and humanities, a disposition is regarded as a quality which manifests itself given suitable circumstances.

And in fact, this was Carnap's procedure which he used in his book "Der logische Aufbau der Welt".

But lateron, he regarded the shortcoming –i.e.: one of the shortcomings– of this form of determining dispositions. For, the statement: "A disposition is regarded as a quality which manifests itself given suitable circumstances" is to be understood as a Total Determination of a Disposition [= TDD], i.e.: a Total Definition of a Dispositional Concept, regarded at first as a unary concept:

 $(TDD) \quad \ \ _{n}\Lambda x \left[x \epsilon F \leftrightarrow \Lambda y \left[\langle x, y \rangle \epsilon Q \rightarrow \langle x, y \rangle \epsilon R \right] \right]^{"}$

But this sentence is logically equivalent to:

as may be seen already by using the Aristotelian laws of contraposition.

Let the universe of discourse consist of objects of one colour; and let b be an object which in past–present–future never ist tested concerning its colour, which means: $\neg \forall y \langle b, y \rangle \in Q^{(*)}$, which logically implies the sent-ence: $\neg \forall y [\langle b, y \rangle \in Q \land \neg \langle b, y \rangle \in R_{red}]^{(*)}$. With regard to TDD' this sentence is analytically equivalent to $\neg b \in \text{Red}^{(*)}$. But $\neg \forall y \langle b, y \rangle \in Q^{(*)}$ also logically implies the statement $\neg \forall y [\langle b, y \rangle \in Q \land \neg \langle b, y \rangle \in R_{green}]^{(*)}$. And with regard to TDD' this sentence is analytically equivalent to $\neg b \in \text{Green}^{(*)}$. But the conjunction $\neg b \in \text{Red} \land b \in \text{Green}^{(*)}$ apriorically contradicts the theory T of the colours of objects which are of one colour.

Therefore, inspite of the fact that TDD is a correct definition w.r.t it form, TDD in inappropriate definition concerning the theory T in question, however T may be formulated in particular.

By the way: Sometimes it was stated that using modal operators will lead to a way out of this problem, i.e.: adding to the universal implication $Ay [\langle x, y \rangle \in Q \rightarrow \langle x, y \rangle \in R]$ " of the definiens of TDD the modal operator of necssity, such that: $\Box Ay [\langle x, y \rangle \in Q \rightarrow \langle x, y \rangle \in R]$ ". But in fact, this is no way out. For *every* modal *logic* is to be in accordance with the metaphysical view of throughout determination, asserting that nothing happens by pure chance, i.e.: that every fact is a necessary fact, so that: $A \leftrightarrow \Box A$ ", so that the modal operator may be regarded as an abbreviation of double negation: $\Box A \leftrightarrow \neg \neg A$ ".

Therefore, we have to look closely to the concept[s] of definition[s] in order to determine how to proceed to get a kind of definition which is correct w.r.t its form as well as w.r.t. its content determined by theory T.

II. On Definitions

The different concepts "definition" are to be defined in the sense of Dubislav as follows: $^{\rm 1}$

Let L [= M^0L] be a language of at least first order whose predicates are at least unary ones. Let T be some theory of M^0L , i.e.: some class of sentences of M^0L , wherby T may be empty or non-empty but finite or denumerable infinite.²

De facto, a definition is always related to some given theory T; for otherwise the concepts of the definiens of the definitions would be without intensions. If T is an exact theory, then the intensions of its concept are exact ones; and of T lacks excatness, then at least some of its concepts will lack exactness, too.³

The background-knowledge of some everyday-language is to be regarded as such a theory which is lacking exactness concerning this and that respect.

³ Take the concept "definition" of everyday English as an example.



¹ Cf. Walter Dubislav "Die Definition" Berlin ³1931, Hamburg ⁴1981; cf. Patrick Suppes "Introduction to Logic" Princeton NJ 1957 (ch. 8); cf. Wilhelm K. Essler "Wissenschaftstheorie I – Definition und Reduktion" Freiburg/München ¹1979, ²1982.

² This statement about M⁰L is formulated by using ML [= M¹L], therby mentioning M⁰L. The statement about M¹L of the preceeding paragraph of this footnote is formulated by using MML [= M²L], thereby mentioning M¹L.

The statement about M²L ...

Definitions may be added to theory T; in this case they are regarded as nominal definitions. Or they may be logically deduced from T; in this case they are seen as real definitions.

Definitions of concepts may be total definitions [= unconditioned definitions] or partial definitions [= conditioned definitions]. Partial definitions may be either strict partional ones or non-strict partional ones; in this case the condition of the definition is entailed by theory T, so that concerning T this definition is in fact a total definition.

The large majority of defined concepts is determined by strict partial definitions. But in textbooks of philosophy, usually –or at least: mainly–total definitions are regarded only. Therefore they here, too, may occupy the first place.

These two criteria determine the concept "total definition": The criterium of eliminability of Blaise Pascal, and the criterium of non-creativity of Stanislaw Leśniewski.⁴

The concept "completely eliminatable" may be defined partially as follows:

Df¹-1: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T]. Then this holds:

 S^n is *completly eliminatable* by A in T concerning N [of T] iff: for every sentence B of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T] there exist a statement E of M⁰L whose extralogical vocabulary consists of concepts of N [of T] such that: $T \cup \{A\} \models B \leftrightarrow E$ "

The concept "non-creative" may be defined partially as follows:

Df¹-2: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T]. Then this holds:

A is *non-creative* concerning S^n w.r.t. N [of T] in T iff:

Concerning the main theorems and their proofs, see: W.K. Essler "Wissenschaftstheorie I" 21982, pp. 93-104



⁴ The other criteria are entailed in these two ones; e.g., the criterium of consistency is entailed in the criterium of non-creativity.

for every sentences *B* and *E* of M⁰L whose extralogical vocabulary consists only of concepts of N [of T] but without S^n this holds: If $T \cup \{A, B\} \models E$, then $T \cup \{B\} \models E$ "

The concept "defined totally" may be defined partially as follows:

Df¹-3: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T]. Then this holds:

 S^n is *defined totally* by A in T concerning N [of T] iff: There exist a sentence D and a set M of n variables $x_1, ..., x_n$ so that the following conditions are satisfied:

(1) *A* is logically equivalent to: $\Lambda x_1 \dots \Lambda x_n [\langle x_1, \dots, x_n \rangle \in S^n \leftrightarrow D]$,

(2) the variables $x_1, ..., x_n$ are different in pairs,

(3) no other variables than $x_1, ..., x_n$ occur free in D,

(4) S^n does not occur in *D*, and

(5) *D* contains –besides bound variables, free variables of the set M, logical constants, and punctuation marks– only ex-pressions of the set N [of T]"

The sentence *A* need not be exactly such a universal equivalence; but it is necessary that *A* is logically equivalent to such a statement.

Th¹-1: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T]. Then this holds:

*S*ⁿ is *defined totally* by *A* in T concerning N [of T] iff:

 S^n is *completly eliminatable* by A in T concerning N [of T], and A is *non-creative* concerning S^n w.r.t. N [of T] in T "

The concept "partially eliminatable" may be defined partially as follows:

Df¹-4: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T]; let C be a sentence of M⁰L whose extralogical vocabulary consists of concepts of N [of T], wherby S^n does not occur in C. Then this holds:

 S^n is *partially eliminatable* by A given C in T concerning N [of T] iff: for every sentence B of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N there exist a statement E of M⁰L whose extralogical vocabulary consists of concepts of N [of T] such that: $T \cup \{A, C\} \models [B \leftrightarrow E]$ "

The concept "non-creative" obviously does not need to be restricted by such a condition *C*.

The concept "defined totally" then may be defined partially as follows:

Df¹-5: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let A be a sentence of M⁰L whose extralogical vocabulary consists of S^n and of concepts of N [of T];]; let C be a sentence of M⁰L whose extralogical vocabulary consists of concepts of N [of T], wherby S^n does not occur in C. Then this holds:

 S^n is *defined partially* by A given C in T w.r.t. N [of T] iff: There exist a sentence D and a set M of n variables $x_1, ..., x_n$ so that the following conditions are satisfied:

(1) *A* is logically equivalent to: $\Lambda x_1 \dots \Lambda x_n [C \rightarrow [\langle x_1, \dots, x_n \rangle \varepsilon S^n \leftrightarrow D]],$

(2) the variables x_1 , ..., x_n are different in pairs,

(3) no other variables than $x_1, ..., x_n$ occur free in D,

(4) S^n does not occur in *D*, and

(5) D contains –besides bound variables, free variables of the set M, logical constants, and punctuation marks– only expressions of the set N [of T]"

Of course, if *C* is deduceable from T then such a partial definition *A* is not a strict one but in fact a total definition.

And if *C* contradicts T, then *A* is not applicatable at all.

Sometimes multiple partial definitions are used, e.g. in arithmetics when a concept is to be defined differently concerning the non-negative numbers and concerning the negative numbers, and e.g. in physics when a concept may be operationalized in different manners. In order to avoid creativity, concerning the different conditions (a) either T will guarantee that these conditions are exclusive ones (b) or we have to make them to be exclusive ones, e.g.: C_0 , $C_1 \land \neg C_0$, $C_2 \land \neg C_1 \land \neg C_0$, In order to receive simplicity at formulating the definition I only will regard the case (a):

Df¹-6: "Let T be a theory of M⁰L; let N be a set of concepts of the vocabulary of theory T; and let S^n be an n-ary concept of M⁰L; let P be a set of m ordered pairs { $\langle A_1, C_1 \rangle$, ..., $\langle A_m, C_m \rangle$ } of sentences of M⁰L, whereby the cognitive vocabulary of each A_i consists of S^n and of concepts of N [of T], and whereby the cognitive vocabulary of each C_i consists of concepts of N [of T]; let T be a theory such that the sentences of the set { $A_1, ..., A_m$ } are mutually exclusive in pairs, according to T. Then this holds:

 S^n is *defined multiple-partially* by the set P of ordered pairs { $\langle A_1, C_1 \rangle$, ..., $\langle A_m, C_m \rangle$ } in T w.r.t. N [of T] iff: For each element $\langle A_i, C_i \rangle$ of P this holds:

 S^n is *defined partially* by A_i given C_i in T w.r.t. N [of T]"

By the way: Suppose that theory T implies both, namely: (1) that the conditions are mutually exclusive in pairs, and (b) that the set of these conditions is exhaustive. Then T implies that this multiple-partial definition is equivalent to a total definition. This may be seen easily in the case of only two conditions C and \neg C, whereby T is the empty set of statements:

Theorem: $[C \to (A \leftrightarrow B)] \land [\neg C \to (A \leftrightarrow E)] \Rightarrow A \leftrightarrow (C \to B) \land (\neg C \to E)$ Theorem: $[C \to (A \leftrightarrow B)] \land [\neg C \to (A \leftrightarrow E)] \Rightarrow A \leftrightarrow (C \land B) \lor (\neg C \land E)$

Note: If the concept of an n-ary function f ist to be defined, then an additional request is needed, nämely: the uniqueness of the value y given an ordered n-tuple $\langle x_1, ..., x_n \rangle$ of objects. If n = 0, then f turns out to be the identity-function, i.e.: the object to be characterized by describing its qualities.

Note: If a theory T grants more than one option to define some concept, only one of them must be taken as the definition of the concept in question and therefore as an analytic truth in T, while the other ones are to be regarded as synthetic [= non-analytic] truths in T.

Note: The statements of theory T may be ordered by a sequence of axioms and theorems, whereby deductive circularities are to be excluded. In the same sense, the concepts of T may be ordered by a sequence of definitions, whereby definitional circularities are to be excluded.

There may be more than one kind of ordering the statements of T by deduction. In the same sense, there may be more than one kind of ordering the concepts of T by definitions.

III: The Problem

At first, Carnap too believed TDD to be the correct form of introducing dispositional predicates. But lateron⁵ he regarded the shortcomming of it w.r.t. non-tested objects. He therefore analysed the procedure of making manifest the dispositions. This analysis may be reconstructed as follows:

"Let Q be the question concerning pairs of objects w.r.t. their equality S, either of colour or of length or

If auch a pair is tested by someone at sometime in the sense of Q, and if the result is positive, then this pair is of quality S; but if it is tested in that way whereby the result is negative, then this pair is not of quality S."

The logical form of such a bilateral reduction sentence is shown by formulas like:⁶

Obviously, BRS is logically equivalent to BRS'.

At a first glance, BRS seems to be a partial definition; but, alas, it does not fit completely into the scheme of partial definitions. Therefore, up to 1971, I tried to develop an alternative scheme concerning dispositional predicates. But years later, I discovered that BRS is creative; for it logically implies –and thus entails– the assumption of universal uniformity.⁷

The concept "universal uniformity" [= "Uu"] is to be understood in this way: "If the result of the test is positive sometimes, then everytime this result of the test is positive":

 $Th^{0}\text{-}1\text{:}\quad \ \ _{n}\Lambda\underline{x}\left[\Lambda\underline{z}\left[\langle\underline{x},\underline{z}\rangle \ \epsilon \ Q \rightarrow \left(\underline{x} \ \epsilon \ S \leftrightarrow \langle\underline{x},\underline{z}\rangle \ \epsilon \ R\right)\right] \rightarrow \underline{x} \ \epsilon \ Uu(Q,R)\right]^{"}$

⁶ In order to shorten the length of the formulas, the following abbreviations are used: x'' for x, y, ..., y',

⁵ See: Rudolf Carnap "Testability & Meaning".

<u>"z</u>" for "<z, t, ... >".

⁷ Cf. W.K. Essler "Wissenschaftstheorie I": ¹1971 versus ²1982.

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Then the question arises whether or not the assumption of universal uniformity and its logical consequences are the only creative ones of BRS, which means, that BRS, restricted to cases of universal uniformity, becomes non-creative:

 $Df^{0}-2: \quad \ \ _{m}\Lambda \underline{x} \left[\underline{x} \ \epsilon \ Uu(Q,R) \rightarrow \Lambda \underline{z} \left[\langle \underline{x},\underline{z} \rangle \ \epsilon \ Q \rightarrow (\underline{x} \ \epsilon \ S \leftrightarrow \langle \underline{x},\underline{z} \rangle \ \epsilon \ R) \right] \right]^{\prime \prime}$

But this question cannot be answered without going back to the TDD. Obviously, TDD is to be restricted to the cases of the test's being carried out. This concept "carry out" [= Co"] is to be defined as follows:

Df⁰-3: $_{,,}\Lambda \underline{x} [\underline{x} \in Co(Q) \leftrightarrow \bigvee \underline{z} (\langle \underline{x}, \underline{z} \rangle \in Q)"$

Then at a first glance the following definition is to be proposed:

 $(\text{CDD}) \quad \ \ _{n}\bigwedge \underline{x} \left[\underline{x} \ \epsilon \ \text{Co}(Q) \rightarrow \left[\underline{x} \ \epsilon \ \text{S} \leftrightarrow \bigwedge \underline{y} \left(\langle \underline{x}, \underline{y} \rangle \ \epsilon \ Q \rightarrow \langle \underline{x}, \underline{y} \rangle \ \epsilon \ R \right) \right] \right]^{"}$

But this restriction ist not sufficient, as it is shown by the following argument: There may be some person b who some day undergoes two times a certain intelligence test; and let the outcomes be very different: The first one may result in: $_{x,\underline{y}_{1}} \in \mathbb{R}^{"}$; and the second one may result in: $_{x,\underline{y}_{2}} \in \mathbb{R}^{"}$, whereby this second result –disregarding the first one–would support the disposition "non-F". But regarding both of them, this proposed definition leads to the contradiction: $_{x} \neg \underline{x} \in \mathbb{F} \land \neg \underline{x} \in \text{non-F}^{"}$. Obviously, this example was constructed by presupposing that there is no universal uniformity available. Therefore, this case, too, is to be excluded:

IV: Its Solution

Then the question arises whether or not Df⁰-2 is the desired solution of the problem of how to determine dispositions und therefore how to define dispositional predicates.

In order to receive an answer, the definitions Df⁰-2 and Df⁰-4 are to be compared. Then answer turns out to be positive:

This indicates:⁸

(a) The definition Df^{0} -2 is correct with regard to ists form; for it is noncreative and yields partial elimination of the concept in question.

(b) The definition Df⁰-4 is correct with regard to its content; for the content of it is formulated by Df⁰-2, which is the bilateral reduction sentence restricted to cases of universal uniformity.

By the way: Instead of Df⁰-4, also the following definition is suitable:

For Df⁰-4 is logically equivalent to Df⁰-5

Th⁰-2:
$$_{,,N\underline{x}} [[\underline{x} \in Co(Q) \land \underline{x} \in Uu(Q,R) \rightarrow [\underline{x} \in S \leftrightarrow \land \underline{y} (\langle \underline{x}, \underline{y} \rangle \in Q \rightarrow \langle \underline{x}, \underline{y} \rangle \in R)]] \leftrightarrow [\underline{x} \in Co(Q) \land \underline{x} \in Uu(Q,R) \rightarrow [\underline{x} \in S \leftrightarrow \forall \underline{y} (\langle \underline{x}, \underline{y} \rangle \in Q \land \langle \underline{x}, \underline{y} \rangle \in R)]]]$$
"

Note: The proof of Th⁰-2 does *not* require the condition $_{,\underline{x}} \in Co(Q)^{"}$ in order to derive these two implications:

Note: BRS is not logically equivalent to CDD; but restricted to the area of universal uniformity, both turn out to be logically euivalent one to another.

Remark: On empirical sciences, definitions w.r.t. the test-result-procedure sometimes are calles "operational definitions" [= "OPD's"].

Therefore, $Df^{0}-4 - as$ well as $Df^{0}-5$, and esp. as well as $Df^{0}-2-$ are the logical form[s] of OPD's. For sentences of this kind, formulates in some background theory, are non-creative; and they indicate how to eliminate the respective concepts within this area of non-creativity.

⁸ Proofs of this theorem as well as related ones may be found in: Wilhelm K. Essler – Rosa F. Martínez Cruzado – Joachim Labude "Grundzüge der Logik I" ⁵2001, ch. IX.



V. Applications

Within the area of logic and mathematics, the concepts are non-ambiguous ones; and this may happen also in some of the theoretical respects of empirical sciences like physics.

But, e.g., in applied physics it some-times turns out to be unavoidable to use concepts whose extensions are somehow blurred at the frontiers of their intensions; and esp. the assumption of universal uniformity is to be used in a blurred manner concerning the value of relative frequency, e.g.: in regarding: "0,98 \approx 1,00" sometimes as: "0,98 = 1,00". For otherwise, physicists would not be able to gain important empirical results.⁹

When new empirical laws are gained, it sometimes may happen that they indicate how to extend the area of the up-to-now-conditions; and then that part of these laws which –according to the background theory T– turn out to be non-creative are added to the up-to-now-definition as a *conservative extension* of it.

Furthermore, it may happen that this conservative extension covers the up-to-now-area, too; and then sometimes –by carrying out some epistemic reorganizing of the Theory T– the new partial definition is regarded as the only one whereby the former one is regarded now as an empirical law.

There are many kinds of dispositions designed by dispositional concepts and described by OPD's. The two main kinds consist (a) of these ones related to a temporal sequence, and in addition (b) of those ones which are not related to some temporal sequence.¹⁰

Ad (a): These are the dispositions which are regarded as being firm during the sequence of time or at least during some period of time, like: *being magnetic,* and: *being highly gifted.*¹¹

Ad (b): These are the observation qualities which are established by resp. perception qualities by using this operational procedure, e.g.:

⁹ The principle of *ex falso quodlibet sequitur* belongs to the fundament of experimental situations and their results.

Without establishing idealizations, there would be no remakable physics.

¹⁰ The concept "time" is used here not in its epistemic [\approx subjective] respect but in its objective respect, i.e.: according to the theory T which is formulated in M⁰L. ¹¹ Don't shorten "are regarded as being" to "are"!

"Some object b is red [at time t]; for some competent person z resp. some measuring apparatus z perceived object b concerning its colour quality at regular circumstances, so that every other observer z' got –or would have got– the same perception as that which z received, namely: some red-perception"

The logical structure of such a procedure is nothing but an OPD.¹²

And, of course, such an OPD is carried out w.r.t. some *used* backgroud theory T by some person z, and it is carried out w.r.t. some *used* measuring theory T by some apparatus z.

If such a person resp. apparatus z were able to reflect its doing, then this z would *mention* that theory T.

In any case: Goethe's statement in his opus "Faust" remains valid: "Am Anfang war die Tat!"

¹² The concepts "perception" and "observation" of M¹L are ussed here not in th vague sense of the ordinary language but in the tchnical jargon of an epistemology which is based on logic including meta-logic, i.e.: which is based on model language philosophy.

